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## LETTER TO THE EDITOR

# Entropy in nonlinear quantum mechanics: II

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**Abstract.** Regarding the discussion of Peres and Weinberg concerning a suitable definition of entropy in nonlinear quantum mechanics, two further observations are made. Firstly, by regarding the covariance matrix of the probability distribution over the phase space of wavefunctions as the nonlinear counterpart ( $\rho_{NL}$ ) of the (linear) density matrix ( $\rho_L$ ) and employing  $-\text{Tr } \rho_{NL} \ln \rho_{NL}$ , one obtains a limiting transition (as nonlinearities vanish), in which this entropy measure converges to the definition in ordinary quantum mechanics,  $-\text{Tr } \rho_L \ln \rho_L$ . Secondly, it is argued that Peres' contention that 'nonlinear variants of Schrödinger's equation violate the second law of thermodynamics' is flawed in that it relies upon the entropy of mixing of non-orthogonal states, which as Dicks and van Dijk have indicated is an undefined concept. A proper approach to associating a quantum mechanical entropy with a mixture of a particle into two non-orthogonal states—by first estimating a suitable two-particle density matrix ( $\tilde{\rho}$ ) and then employing  $-\text{Tr } \tilde{\rho} \ln \tilde{\rho}$ —is outlined.

In a previous letter [1], I sought to 'mediate' the discussion of Peres [2] and Weinberg [3] concerning the issue of whether or not entropy could be suitably defined in Weinberg's formulation [4] of nonlinear quantum mechanics. Relying upon Guiasu's concept of a 'probability space of wavefunctions' [5], I suggested (as Weinberg had [3]) that the measure be taken as the entropy of the probability distribution over the 'phase space' of wavefunctions. As nonlinearities vanish, this would reduce to the entropy associated with the linear quantum mechanical density matrix ( $\rho_L$ ), viewing  $\rho_L$  as Guiasu did [5], as the covariance matrix of a (zero-mean) complex multinormal distribution over the space of wavefunctions. This entropy could be written as

$$p + p \ln 2\pi + \frac{1}{2} \ln(\det \Sigma_0) \quad (1)$$

where  $p$  is the dimension of  $\rho_L$  and  $\Sigma_0$  is a  $2p$ -dimensional real covariance matrix associated with the  $p$ -dimensional complex covariance matrix  $\rho_L$ .

Here I would like to observe that a less 'radical' reconciliation could be achieved by in fact maintaining the von Neumann definition,  $-\text{Tr } \rho \ln \rho$ , of the entropy in both the linear and nonlinear cases, using for  $\rho$  in the nonlinear case, the covariance matrix (in some suitable orthogonal basis) of the probability distribution over the phase space of wavefunctions. With vanishing nonlinearities,  $\rho_L$  is a sufficient statistic for estimating—following Guiasu [5]—the zero-mean (complex multinormal) probability distribution over phase space. (In the nonlinear instance, information in addition to  $\rho_{NL}$  is needed to determine the (non-Gaussian) probability distribution. Ordinary (linear) quantum mechanics can then be viewed as the Gaussian case of nonlinear quantum mechanics.)

In his comment 'Nonlinear variants of Schrödinger's equation violate the second law of thermodynamics', Peres [2] had devised an argument based on the (von Neumann) entropy of a density matrix representable as a mixture of two (in general, *non-orthogonal*) states. However, Dieks and van Dijk [6] have argued that 'an entropy of mixing is *not defined* if the quantum states of the particle are no longer orthogonal . . . . It is possible for some purposes to replace the mixture of non-orthogonal components by a mixture in which the components *are* orthogonal and, which, as a consequence, *has* an entropy of mixing. The original mixture and its thus-defined substitute are, however, not equivalent in all thermodynamically relevant respects'.

In another communication [7], entitled 'Maximum-entropy estimation of a two-particle density matrix from a mixture of a particle into two non-orthogonal states', I have developed upon Dieks and van Dijk's argument. I contend that the (von Neumann) entropy to be associated with a mixture of a particle into two non-orthogonal states (following the notation of Peres [2])

$$\rho = \lambda P_\phi + (1 - \lambda) P_\psi \quad (2)$$

(where  $0 < \lambda < 1$  and  $P_\phi$  and  $P_\psi$  are projection operators on the pure states  $\phi$  and  $\psi$ , respectively) is *not* properly (as Peres assumes and Dieks and van Dijk [6] reject)

$$S = -k \sum_{j=1}^2 \omega_j \ln \omega_j \quad (3)$$

where

$$\omega_j = \frac{1}{2} \pm \left[ \frac{1}{4} - \lambda(1 - \lambda)(1 - x) \right]^{1/2} \quad (4)$$

are the non-vanishing eigenvalues of  $\rho$  and  $x = |\langle \phi | \psi \rangle|^2$ . Instead, I propose that one needs to first solve the (two-particle) maximum-entropy estimation problem [cf 8-10]

$$\text{maximize } -\text{Tr } \tilde{\rho} \ln \tilde{\rho} \quad (5)$$

subject to the constraints on transition probabilities ( $\otimes$  is the tensor product)

$$\text{Tr}(\tilde{\rho}(P_\phi \otimes P_\psi)) = \lambda(1 - \lambda) \quad (6)$$

$$\text{Tr}(\tilde{\rho}(P_\psi \otimes P_\phi)) = \lambda(1 - \lambda) \quad (7)$$

$$\text{Tr}(\tilde{\rho}(P_\phi \otimes P_\phi)) = \lambda^2 \quad (8)$$

$$\text{Tr}(\tilde{\rho}(P_\psi \otimes P_\psi)) = (1 - \lambda)^2 \quad (9)$$

as well as constraints requiring that the partial traces of  $\tilde{\rho}$  over each particle yield  $\rho$ . (In [7], the argument is presented more formally using a general framework [11] for analysing compounds of two quantum systems.) The value of (5) is then the best estimate of the (von Neumann) entropy to be assigned to the non-orthogonal mixture. (If  $\phi$  and  $\psi$  are, in fact, orthogonal, the value of (5) is twice the value of (3) [12].)

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